Problem 1 (40 per cent)

True, false, uncertain. Explain your answer! You can atmost get half points for a correct answer without explanation.

1. The HO model is consistent with the rise in the skillpremium in the United States since 1980.

Answer: True. The United States is skill-abundant and consequently the Stolper Samuelson theorem tells us that skilled workers will benefit from trade.

2. Exporters are more productive than non-exporters. This is entirely due to more productive firms choosing to become exporters.

Answer: False. De Loecker shows this is in part due to learning-by-exporting.

3. The infant-industry argument is theoretically coherent but empirically dubious.

Answer: True. With increasing external returns to scale, trade protection can potentially shift the economy to a higher welfare equilibrium. In practice, this is difficult to do. Brazil and India largely failed, wheras some argue Korea is a positive case of infant-industry protection. Students also get full points if they take a stand and argue that, say, Korea proved the point.

4. Subsidizing exports is never an optimal government strategy

Answer: False. In the Cournot model it is always optimal to subsidize to some extent. Under certain conditions you can even make that argument in a model with external returns to scale.

Problem 2 (60 per cent).

Consider a Krugman model of a single economy with only one factor of production, labor. Total stock of labor is L. There is a representative agent with utility:

$$U = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where N is the number of products in the economy, c_i is consumption of good i and $\sigma > 1$.

Each product can be produced by a single monopolist. The labor cost of producing $y > 0$ units is

$$l = \alpha + \beta y,$$

where $\alpha, \beta > 0$ and unit labor costs required for producing zero units is zero. We denote by w the wage, i.e. factor payments to one unit of labor. The labor cost of producing is identical for all firms.

Question 1. Derive the demand curve for a single variety, i taking total income wL as given. Show that it equals:

$$c_i = \frac{wL}{P} \left(\frac{p_i}{P} \right)^{-\sigma},$$

where P is the ideal price index defined as:

$$P^{1-\sigma} = \sum_{i=1}^{N} p_i^{1-\sigma}$$

Hint: Utility is ordinal. Hence, it is possible (though not required) to maximize $\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}$ instead.

Answer: We maximize utility by setting up the problem:

$$\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \lambda\left(\sum_{i=1}^{N} p_i c_i - wL\right),$$

which gives:

$$c_i^{-1/\sigma}\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \lambda p_i$$

I solve for λ by multiplying both sides by c_i:

$$c_i^{\frac{\sigma-1}{\sigma}}\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \lambda p_i c_i$$

and then sum over all products to get:

$$\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \lambda wL \Leftrightarrow$$

$$\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} = \lambda wL$$

I then take the expression from above to get:

$$c_i^{-1/\sigma}\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \lambda p_i \Leftrightarrow$$

$$c_i^{\frac{\sigma-1}{\sigma}}\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{-1} = \lambda^{1-\sigma} p_i^{1-\sigma}.$$

And then sum to get:

$$1 = \lambda^{1-\sigma}\sum_{i=1}^{N} p_i^{1-\sigma},$$

which then implies:

$$c_i^{-1/\sigma}\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \lambda p_i,$$

which we reduce to get:

$$c_i^{-1/\sigma}\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = \lambda p_i \Leftrightarrow$$

$$c_i\left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}}\right)^{-\frac{\sigma}{\sigma-1}} = \lambda^{-\sigma} p_i^{-\sigma} \Leftrightarrow$$

$$c_i = \lambda w L \lambda^{-\sigma} p_i^{-\sigma} = \lambda^{1-\sigma} w L p_i^{-\sigma} \Leftrightarrow$$

$$c_i = P^{1-\sigma} w L p_i^{-\sigma} = \frac{wL}{P}\left(\frac{p_i}{P}\right)^{-\sigma}.$$

Which is we set out to prove.

A given firm takes this demand function as given but can choose p_i.

Question 2. Solve for the firm's optimal price.

Answer: This is a familiar markup problem so the solution is:

$$p = \frac{\sigma}{\sigma-1}\beta w.$$

Let there be free entry such that equilibrium profits equal zero for each firm.

Question 3. What is equilibrium production for each firm?

Answer: Equilibrium production requires zero profits, such that:

$$\left(\frac{\sigma}{\sigma-1}\beta w - \frac{\sigma-1}{\sigma-1}\beta w\right) y - \alpha w = 0 \Leftrightarrow$$

$$\frac{1}{\sigma-1}\beta y = \alpha \Leftrightarrow$$

$$y = \frac{\alpha}{\beta}(\sigma-1).$$

Question 4. How does equilibrium production depend on α β and σ? Give economic intuition for each.

Answer: Higher fixed costs, α, requires higher production to obtain zero profits. Higher marginal costs, β, implies higher prices and higher profit per unit sold (due to markup pricing) and therefore requires fewer units for zero profit. Finally, the elasticity of substitution, σ, gives the level of competition and therefore prices. Higher σ means lower markup and therefore higher required y. These things have to be properly explained to give full marks.

Question 5.

Find the equilibrium number of firms.

Answer: The labor market clearing condition serves us well:

$$L = (\alpha + \beta y)N \Leftrightarrow$$

$$L = (\alpha + \alpha(\sigma-1))N \Leftrightarrow$$

$$N = \frac{L}{\alpha\sigma}$$

Question 6. Derive the utility of a representative agent as a function only of exogenous parameters.

Answer: The utility function, using the symmetry of the model, gives:

$$U = \left(\sum_{i=1}^{N} c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} = N^{\frac{\sigma}{\sigma-1}} c,$$

where c is idential consumption per person of each variety. Production per firm is y and therefore consumption per person is:

$$c = \frac{y}{L} = \frac{\alpha}{L\beta}(\sigma - 1),$$

Such that:

$$U = \left(\frac{L}{\alpha\sigma} \right)^{\frac{\sigma}{\sigma-1}} \frac{\alpha}{L\beta}(\sigma - 1) = L^{\frac{1}{\sigma-1}} \sigma^{-\frac{\sigma}{\sigma-1}} (\sigma - 1) \alpha^{\frac{-1}{\sigma-1}} / \beta.$$

Question 7. So far, we have been talking about a closed model. How would I conclude anything about the gains from trade in this model? What would be the source of these gains?

Answer: We have talked about this in class a hundred times so they need to get this right. In a model with no trading cost doubling the population is the same as two countries of equal size trading with one another, i.e. I can consider Denmark and Sweden as one economy and derive the gains from trade by Sweden and Denmark trading by considering the economy of a combined Denmark and Sweden, i.e. a higher population. Hence, I can see that utility is increasing in L and conclude that there are gains from trade. In this model the sources of gains from trade are from increased variety and only that.

Question 8. Name an additional source of gains from trade from Krugman's original article. What would I need to change in this model to get that source of gains from trade?

Answer: *Krugman originally had exploitation of scale as well, in that firms produce more goods and thereby spread fixed cost on more products. In addition, he has lower prices. The latter is not a true general equilibrium gain but is acceptable for this exam. In either case we need to make σ dependent on c, in particular $\partial\sigma/\partial c < 0$.*

These days, it is often argued that the Covid-19 pandemic implies that we should scale back globalization and potentially protect our markets more. Now consider two equally sized countries, each with a population of 1, called A and B. They can each set a tariff, τ^A and τ^B. These are gross tariffs so when $\tau^A = 1$ there is no tariff and when $\tau^A > 1$ the government collects $\tau^A - 1$ in tariffs. We will only consider equilibria in which both countries set the same tariff: $\tau = \tau^A = \tau^B$. We will examine the effects of tariffs. (Note, whatever change to the model you suggested in Question 8 is *not* relevant here)

Question 9. Show that utility of a representative agent, when both countries set a tariff of $\tau \geq 1$ is:

$$U^{\frac{\sigma-1}{\sigma}} = \frac{\sigma-1}{\sigma}\frac{1}{\beta}\frac{\left[1+\tau^{\sigma-1}\right]}{\tau^{\sigma}+1},$$

and that production per firm is:

$$y = \frac{\alpha}{\beta}(\sigma-1).$$

Answer: This is difficult so an answer that is largely correct should be given full points. Demand elasticity continues to be σ so firms continue to set price as a markup. However, with the tariff the prices at home and foreign for the consumer are, respectively:

$$p^{home} = p^{H} = \frac{\sigma}{\sigma-1}\beta$$

$$p^{foreign} = p^{F} = \frac{\sigma}{\sigma-1}\beta\tau,$$

which is symmetric for both country A and B. Each firm still has a zero profit condition of:

$$\left(\frac{\sigma}{\sigma-1}\beta-\beta\right)wy - w\alpha = 0 \Leftrightarrow$$

$$y = \frac{\alpha}{\beta}(\sigma-1),$$

so production of each firm is the same as in the closed economy. With it, we get number of firms of:

$$N(\alpha+\beta y) = 2 \Leftrightarrow$$

$$N = \frac{2}{\alpha+\beta y} = \frac{2}{\alpha+\alpha(\sigma-1)} = \frac{2}{\alpha\sigma},$$

with half in each country. Symmetry implies that a consumer consumes equally of all home goods. And also consumes equally of all foreign goods.

Further, relative consumption of the two types is:

$$c^{H}/c^{F} = \left(p^{H}/p^{F}\right)^{-\sigma} = \tau^{\sigma}.$$

and it must be true that:

$$c^{H}+c^{F} = y = \frac{\alpha}{\beta}(\sigma-1) \Leftrightarrow$$

$$c^{F}\left(\tau^{\sigma}+1\right) = \frac{\alpha}{\beta}(\sigma-1) \Leftrightarrow$$

$$c^F = \frac{\alpha}{\beta}\frac{(\sigma - 1)}{\tau^\sigma + 1}.$$

And therefore a representative agent has utility:

$$\begin{aligned} U^{\frac{\sigma-1}{\sigma}} &= \frac{N}{2}\left(c^F\right)^{\frac{\sigma-1}{\sigma}} + \frac{N}{2}\left(c^H\right)^{\frac{\sigma-1}{\sigma}} \\ &= \frac{N}{2}\left(c^F\right)^{\frac{\sigma-1}{\sigma}}\left[1 + \left(c^H/c^F\right)^{\frac{\sigma-1}{\sigma}}\right] \\ &= \frac{N}{2}\left(\frac{\alpha}{\beta}\frac{(\sigma-1)}{\tau^\sigma+1}\right)^{\frac{\sigma-1}{\sigma}}\left[1+\tau^{\sigma-1}\right] \\ &= \frac{1}{\alpha\sigma}\left(\frac{\alpha}{\beta}(\sigma-1)\right)^{\frac{\sigma-1}{\sigma}}\frac{\left[1+\tau^{\sigma-1}\right]}{(\tau^\sigma+1)^{\frac{\sigma-1}{\sigma}}} \end{aligned}$$

Which is what we were looking for.

Question 10. Show that world welfare is highest when $\tau = 1$ (don't worry about second order conditions)

Answer: Take logs of utility and differentiate to get:

$$\frac{\sigma-1}{\sigma}logU = log\left[\frac{1}{\alpha\sigma}\left(\frac{\alpha}{\beta}(\sigma-1)\right)^{\frac{\sigma-1}{\sigma}}\right] + log\left[1+\tau^{\sigma-1}\right] - \frac{\sigma-1}{\sigma}log(\tau^\sigma+1)$$

which then gives:

$$\frac{\sigma-1}{\sigma}\frac{\partial logU}{\partial\tau} = \frac{(\sigma-1)\tau^{\sigma-2}}{1+\tau^{\sigma-1}} - \frac{(\sigma-1)\tau^{\sigma-1}}{\tau^\sigma+1}$$

and that equals zero at $\tau = 1$.

Question 11. How do tariffs distort international trade? Give an economic interpretation of the margin(s) of distortion.

Answer: The distortion does not come from a change in the number of varieties: each consumer still consumes the full set of varieties. Consequently, it also does not come from a distortion in how much is produced since each firm continues to produce $y = \frac{\alpha}{\beta}(\sigma-1)$. *However, the tariff introduces a distortion in the amount consumed. We have* $c^F/c^H = \tau^\sigma$ *whereas the efficient relative consumption equals 1.*